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Universality of the shear viscosity in supergravity

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Abstract

Kovtun, Son and Starinets proposed a bound on the shear viscosity of any fluid in terms of its entropy density. We argue that this bound is always saturated for gauge theories at large 't Hooft coupling, which admit holographically dual supergravity description.

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1 Introduction

One of the remarkable connections arising from holography has been the link between black hole thermodynamics and the more traditional case on the field theory side. By working with a black hole (or black brane) background on the gravity side, this allows the investigation of the corresponding gauge theory at finite temperature. Such a connection alone has already yielded many new insights on the thermal phase structure of gauge theories. On the other hand, it is important to realize that basic equilibrium thermodynamic quantities, such as the free energy and entropy do not provide complete information about the theory. In principle, with an exact AdS/CFT prescription, it ought to be possible to provide dual descriptions of any desired process in the gauge theory.

In practice, of course, we do not expect to find a simple description encompassing all of the information of the gauge theory. However, in keeping with thermodynamic ideas, it is natural to expect that the long-distance fluctuations in the theory will have a hydrodynamic description. In this manner, one may expand the study of gauge theories at finite temperature to encompass, *e.g.*, transport phenomenon such as diffusion and sound propagation [1, 2, 3, 4, 5]. Along these lines, Kovtun, Son and Starinets (KSS) [6] extended the previous results of [1, 2, 3] and investigated the shear viscosity, η , for a large variety of backgrounds.

In the examples of [6], which cover all maximally supersymmetric gauge theories and $\mathcal{N} = 2^*$ gauge theory (to leading order in m/T) [7, 8], it was found that the ratio of shear viscosity η to the entropy density s had a fixed value, $\eta/s = 1/4\pi$. On the other hand, it can be shown that coupled systems have $\eta/s \gg 1$, and even common substances have ratios well above this value, which upon reintroducing fundamental constants becomes

$$\frac{1}{4\pi} \rightarrow \frac{\hbar}{4\pi k_B} \approx 6.08 \times 10^{-13} \text{ K} \cdot \text{s}. \quad (1.1)$$

Based on these observations, KSS conjectured that there is a universal bound in nature for this ratio, namely [6]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}. \quad (1.2)$$

It was further argued in [9] that this bound follows from the generalized covariant entropy bound [10].

The intriguing result that $\eta/s = 1/4\pi$ holds exactly for many different nonextremal brane backgrounds suggests that saturation of the bound (1.2) may always be true for

systems admitting a dual supergravity realization. In this letter, we demonstrate that this is in fact always the case. Before doing this however, we provide a brief review of the hydrodynamics of strongly coupled systems in section 2. Then in section 3 we point out that the bound (1.2) is saturated for $\mathcal{N} = 2^*$ gauge theory [7], Klebanov-Tseytlin (KT) gauge theory [11], and Maldacena-Nunez (MN) gauge theory [12]. This leads us, at the end of the section, to prove that the bound is always saturated in the supergravity approximation of strongly coupled gauge theories.

Of course, as nature has demonstrated, the shear viscosity bound is not necessarily saturated at weak coupling. This, however, is not in contradiction with our proof, as one cannot directly compare strong and weak coupling results. This was already been seen in a different context in *e.g.* the case of the entropy of $\mathcal{N} = 4$ super-Yang-Mills theory at weak and strong coupling. Nevertheless, the conjecture that there is a minimum value of η/s still remains to be tested. In particular, in order to verify the validity of (1.2) in the framework of gauge/string correspondence, one has to go beyond the supergravity approximation and include α' corrections (corresponding to finite 't Hooft coupling corrections). We consider some aspects of this issue in section 4.

2 Hydrodynamics and diffusion in supergravity duals

Just as in thermodynamics, hydrodynamics is not concerned with the microscopic properties of a theory, but instead in its macroscopic ones. Overall, hydrodynamics may be invoked to provide an effective description of long wavelength and long time properties of a macroscopic medium. In this section, we briefly review a few key ideas in the application of hydrodynamics to the study of strongly coupled gauge theories.

Of particular interest to hydrodynamics is the study of diffusion governing the flow of say heat or charge through a medium. Assuming we have a charge related to a conserved current, the diffusion of the charge is then governed by its local concentration, so that $\vec{j} = -D\vec{\nabla}j^0$. Combining this with current conservation (also thought of as the continuity equation), $\partial_t j^0 + \vec{\nabla} \cdot \vec{j} = 0$, then yields the familiar heat equation, $\partial_t j^0 = \vec{\nabla} \cdot (D\vec{\nabla}j^0)$. As expected for a thermodynamic description, these equations are no longer Lorentz invariant, and time reversal invariance is explicitly broken.

While this is all well known, its application to AdS/CFT is perhaps less familiar. The important point here is that the diffusion coefficient \mathcal{D} is connected to the underlying properties of the gauge theory. At the same time, techniques have been

developed to extract \mathcal{D} from the fluctuations of long-wavelength modes in the supergravity dual [1, 2, 3, 6]. So for strongly coupled gauge theories where the dual is known, computation of \mathcal{D} and other kinetic coefficients yields additional insight on the nature of the theory itself.

Of more direct concern to us is bulk transport through a medium. In this case, one works with energy, momentum and pressure, or in other words a conserved stress-energy tensor with components T^{00} , T^{0i} and T^{ij} . While the analysis is similar to that of charge diffusion, some additional complication arises from the tensor nature of $T^{\mu\nu}$. The resulting hydrodynamic quantities of interest include the bulk viscosity ζ , shear viscosity η , and the speed of sound v_s .

In order to compute these kinetic coefficients from the gravity dual, one may in principle extract the appropriate behavior of the boundary stress tensor $T^{\mu\nu}$. Alternatively, as demonstrated in [6], the shear viscosity may be determined by setting up a ‘shear perturbation’ as a fluctuation on top of the original supergravity background, given by the metric

$$ds^2 = (G_{00}(u)dX_0^2 + G_{xx}(u)d\vec{x}^2) + G_{uu}(u)du^2 + \cdots, \quad (2.1)$$

where the dual gauge theory has (X_0, \vec{x}) coordinates, u is the transverse coordinate, and the ellipses denote compact directions which are not of direct concern in the following. This metric has a plane-symmetric horizon that extends in p infinite spatial directions, $\vec{x} = \{x^i\}$. We assume this metric has a horizon at $u \rightarrow u_0$ where G_{00} vanishes. The decay of this shear mode is then governed by a diffusion coefficient

$$\mathcal{D} = \frac{\sqrt{-G(u_0)}}{\sqrt{-G_{00}(u_0)G_{uu}(u_0)}} \int_{u_0}^{\infty} du \frac{-G_{00}G_{uu}}{G_{xx}\sqrt{-G}}, \quad (2.2)$$

denoted the shear mode diffusion constant in [6].

The shear viscosity, η , may be extracted from the diffusion constant \mathcal{D} according to [6]

$$\mathcal{D} = \frac{\eta}{\epsilon + P} = \frac{1}{T} \frac{\eta}{s}, \quad (2.3)$$

in the dual gauge theory. Here ϵ , s , P and T are correspondingly the equilibrium energy and entropy densities, the pressure and temperature. As a result, the conjectured shear viscosity bound, (1.2), is equivalent to the statement

$$\mathcal{D} \geq \frac{1}{4\pi T}. \quad (2.4)$$

This is the form of the bound that we use in the subsequent sections.

3 Applications

In this section we compute the shear diffusion constant for a class of supergravity backgrounds realizing supergravity duals to four dimensional gauge theories with eight or less supercharges, namely the $\mathcal{N} = 2^*$ Pilch-Warner (PW) solution [7], the supergravity dual to the Klebanov-Tseytlin cascading gauge theory (KT) [11], and the $\mathcal{N} = 1$ Maldacena-Nunez solution [12]. In all cases we find that the KSS bound, (1.2), is saturated. We then prove that it is always saturated in the supergravity approximation to gauge theories at strong 't Hooft coupling.

3.1 $\mathcal{N} = 2^*$ gauge theory

The supergravity dual to $\mathcal{N} = 2^*$ $SU(N)$ gauge theory has been proposed in [7]. The nonextremal deformation of the PW solution has been studied in [8]. This solution realizes the supergravity dual to $\mathcal{N} = 4$, $SU(N)$ gauge theory softly broken to $\mathcal{N} = 2$. The high temperature thermodynamics of this system thus involves a small parameter $\xi \equiv m/T$, where m is the mass of the $\mathcal{N} = 2$ hypermultiplet, giving rise to $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ partial supersymmetry breaking. That the KSS bound is saturated in this system to leading order in ξ has been observed in [6]. Here, we show that it is actually valid for arbitrary ξ . We will use an exact five-dimensional description of the black hole, although since its 10-d lift is known [8], the same result can be obtained directly in ten dimensions.

The relevant near-extremal 5-d Einstein frame metric involves two functions A , B of a radial coordinate r

$$ds_5^2 = e^{2A} (e^{2B} dX_0^2 + d\vec{x}^2) + dr^2. \quad (3.1)$$

The horizon is taken to be at $r_{hor} = 0$. One of the equations of motion is (Eq. (3.19) of [8])

$$\ln B' + 4A + B = 4\alpha + \ln \delta, \quad (3.2)$$

where $\{\alpha, \delta\}$ are constants specified by the near-horizon asymptotics of A , B (eq. (3.20) of [8])

$$\begin{aligned} A &\rightarrow \alpha, \\ e^B &\rightarrow \delta r. \end{aligned} \quad (3.3)$$

Notice that we can rewrite (3.2) as

$$(e^{2B})' = 2\delta e^{4\alpha} e^{-4A+B}. \quad (3.4)$$

Now we can use (2.2) to compute the shear diffusion constant

$$\begin{aligned} \mathcal{D}_{PW} &= e^{3A} \Big|_{horizon} \int_0^{+\infty} dr e^{-4A+B} \\ &= e^{3A} \Big|_{horizon} \int_0^{+\infty} d(e^{2B}) \frac{1}{2} \delta^{-1} e^{-4\alpha} \\ &= \frac{1}{2} \delta^{-1} e^{-\alpha}. \end{aligned} \quad (3.5)$$

Since the black hole temperature is [8]

$$T = \frac{1}{2\pi} \delta e^{\alpha}, \quad (3.6)$$

we conclude that

$$\mathcal{D}_{PW} = \frac{1}{4\pi T}, \quad (3.7)$$

which generalizes the result of [6] to all orders in ξ .

3.2 KT gauge theory

The supergravity dual to $\mathcal{N} = 1$ cascading $SU(N+M) \times SU(N)$ gauge theory has been proposed in [11, 13]. The nonextremal deformations of the KT solution [11] has been studied in [14, 15, 16]. In this section we follow [16].

The relevant near-extremal 10-d Einstein-frame metric involves four functions x, y, z, w of a radial coordinate u

$$\begin{aligned} ds_{10E}^2 &= e^{2z} (e^{-6x} dX_0^2 + e^{2x} d\vec{x}^2) + e^{-2z} ds_6^2, \\ ds_6^2 &= e^{10y} du^2 + e^{2y} (dM_5)^2, \\ (dM_5)^2 &= e^{-8w} e_\psi^2 + e^{2w} (e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2), \\ e_\psi &= \frac{1}{3} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2), \quad e_{\theta_i} = \frac{1}{\sqrt{6}} d\theta_i, \quad e_{\phi_i} = \frac{1}{\sqrt{6}} \sin \theta_i d\phi_i. \end{aligned} \quad (3.8)$$

Here X_0 is the euclidean time and \vec{x} are the three longitudinal D3-brane directions. Also, with our choice of the radial coordinate, the horizon is at $u = +\infty$, and the boundary is at $u = 0$.

The system of equations governing the solution is

$$\begin{aligned}
x'' &= 0, \quad x = au, \quad a = \text{const} > 0, \\
10y'' - 8e^{8y}(6e^{-2w} - e^{-12w}) + \Phi'' &= 0, \\
10w'' - 12e^{8y}(e^{-2w} - e^{-12w}) - \Phi'' &= 0, \\
\Phi'' + e^{-\Phi+4z-4y-4w}(f'^2 - e^{2\Phi+8y+8w}P^2) &= 0, \\
4z'' - (Q + 2Pf)^2 e^{8z} - e^{-\Phi+4z-4y-4w}(f'^2 + e^{2\Phi+8y+8w}P^2) &= 0, \\
(e^{-\Phi+4z-4y-4w}f')' - P(Q + 2Pf)e^{8z} &= 0.
\end{aligned} \tag{3.9}$$

The integration constants are subject to the zero-energy constraint $T + V = 0$, *i.e.*

$$\begin{aligned}
y'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w}f'^2 \\
- e^{8y}(6e^{-2w} - e^{-12w}) + \frac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \frac{1}{8}e^{8z}(Q + 2Pf)^2 = 3a^2.
\end{aligned} \tag{3.10}$$

Now we can evaluate the shear diffusion constant for the nonextremal KT solution. Following (2.2), we find

$$\begin{aligned}
\mathcal{D}_{KT} &= e^{5y+3x-2z} \Big|_{\text{horizon}}^{\text{boundary}} \int_{\text{horizon}}^{\text{boundary}} du e^{-8x} \\
&= e^{5y+3x-2z} \Big|_{\text{horizon}}^{\text{boundary}} \int_{\text{horizon}}^{\text{boundary}} du e^{-8au} \\
&= \frac{1}{8a} e^{5y+3x-2z} \Big|_{\text{horizon}}.
\end{aligned} \tag{3.11}$$

The asymptotics of the solution to (3.9) at the horizon, $u \rightarrow +\infty$, are [16]

$$\begin{aligned}
z &\rightarrow -au + z_*, \\
y &\rightarrow -au + y_*, \\
x &= au.
\end{aligned} \tag{3.12}$$

Thus (3.11) evaluates to

$$\mathcal{D}_{KT} = \frac{1}{8a} e^{5y_*-2z_*}. \tag{3.13}$$

Since the black hole temperature is [16]

$$T = \frac{2}{\pi} a e^{2z_*-5y_*}, \tag{3.14}$$

we indeed find

$$\mathcal{D}_{KT} = \frac{1}{4\pi T}. \tag{3.15}$$

3.3 MN gauge theory

The supergravity dual to $\mathcal{N} = 1$ $SU(N)$ supersymmetric Yang-Mills theory has been proposed in [12]. The nonextremal deformation of the MN solution has been studied in [17, 18]. In this section we follow [17]. The relevant near-extremal 10-d Einstein-frame metric is

$$ds_E^2 = c_1(r)^2 [\Delta_1^2 dX_0^2 + d\vec{x}^2] + c_1(r)^2 a^2 \left[\frac{dr^2}{\Delta_2^2 r^2} + \frac{h(r)}{4} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + \frac{1}{4} \left(d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 \right], \quad (3.16)$$

One of the equations of motion is (eq. (5.55) of [17])

$$\Delta_1' \Delta_2 = \frac{A}{c_1(r)^8 h(r) r}, \quad (3.17)$$

where (a constant) A is the nonextremality parameter. Now we can compute

$$\begin{aligned} \mathcal{D}_{MN} &= a^5 h c_1^8 \Big|_{horizon}^{\int_{horizon}^{boundary} dr \frac{\Delta_1}{\Delta_2 c_1^8 h r}} \\ &= a^5 h c_1^8 \Big|_{horizon}^{\int_{horizon}^{boundary} \frac{1}{2Aa^4} d[\Delta_1^2]} \\ &= a^5 h c_1^8 \Big|_{horizon}^{\frac{1}{2Aa^4}}, \end{aligned} \quad (3.18)$$

where in the second line in (3.18) we used (3.17), and in the last line we used the boundary conditions $\Delta_1|_{horizon} = 0$ and $\Delta_1|_{boundary} = 1$.

From the near horizon behavior of the metric (eq. (5.61) of [17])

$$\begin{aligned} ds_E^2 &\approx c_1(r_h)^2 [\eta^2 dX_0^2 + d\vec{x}^2] + \frac{c_1^{18}(r_h) a^2 h^2(r_h)}{A^2} d\eta^2 \\ &+ c_1(r_h)^2 a^2 \left[\frac{h(r_h)}{4} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + \frac{1}{4} \left(d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 \right], \end{aligned} \quad (3.19)$$

we find

$$\frac{1}{T} = 2\pi \frac{c_1^8 a h}{A} \Big|_{horizon}. \quad (3.20)$$

Comparing (3.18) and (3.20) we find

$$\mathcal{D}_{MN} = \frac{1}{4\pi T} . \quad (3.21)$$

Hence for at three non-trivial supergravity backgrounds we have found a common result, that $\mathcal{D} = 1/4\pi T$. In all cases, we have taken advantage of reducing the u integral in (2.2) to a boundary term. This suggests that saturation of the KSS bound is in fact universal, depending only on the thermal nature of the background spacetime. We now examine this in detail.

3.4 General supergravity backgrounds

Our key observation is that before turning on any nonextremality, the Poincare symmetry of the background geometry guarantees that

$$R_{tt} + R_{xx} = 0 , \quad (3.22)$$

where $R_{\mu\nu}$ is the Ricci tensor in an orthonormal frame. Clearly, an analogous condition must be satisfied for the full stress tensor of the matter supporting the geometry

$$T_{tt} + T_{xx} = 0 . \quad (3.23)$$

Because turning on the nonextremality will not modify (3.23), we see that (3.22) is valid away from extremality as well. We now show that (3.22) is sufficient to explicitly evaluate (2.2), yielding the result

$$\mathcal{D}T = \frac{1}{4\pi} . \quad (3.24)$$

Consider the following $D = d + p + q$ dimensional background

$$ds_D^2 = \Omega_1^2(y) (g_{\mu\nu}(x) dx^\mu dx^\nu) + \Omega_2^2(y) (\hat{g}(z)_{\alpha\beta} dz^\alpha dz^\beta + \tilde{g}(y)_{mn} dy^m dy^n) , \quad (3.25)$$

where $g_{\mu\nu}$ is d -dimensional, $\hat{g}_{\alpha\beta}$ is p -dimensional and g_{mn} is q -dimensional. An explicit computation yields

$$\begin{aligned} R_{\mu\nu} &= r_{\mu\nu} - g_{\mu\nu} \left(\Omega_2^{-2} \Omega_1 \nabla^2 \Omega_1 + (d-1) \Omega_2^{-2} (\nabla \Omega_1)^2 + (D-d-2) \Omega_2^{-3} \Omega_1 \nabla \Omega_1 \nabla \Omega_2 \right) , \\ R_{\alpha\beta} &= \hat{r}_{\alpha\beta} - \hat{g}_{\alpha\beta} \left(\Omega_2^{-1} \nabla^2 \Omega_2 + (D-d-3) \Omega_2^{-2} (\nabla \Omega_2)^2 + d \Omega_2^{-1} \Omega_1^{-1} \nabla \Omega_1 \nabla \Omega_2 \right) , \end{aligned} \quad (3.26)$$

where ∇ is with respect to g_{mn} , and $r_{\mu\nu}, \hat{r}_{\alpha\beta}, \tilde{r}_{mn}$ are Ricci tensors computed from $g_{\mu\nu}, \hat{g}_{\alpha\beta}$ and \tilde{g}_{mn} correspondingly.

To be relevant for the nonextremal RG flows of 4-d gauge theories, we now take $d = 1$, $p = 3$ and $D = 10$. Also, we have $r_{\mu\nu} = \hat{r}_{\alpha\beta} = 0$, and $\Omega_1(r) = \Omega_2(r)\Delta(r)$ depends only on the radial coordinate of \tilde{g}_{mn} . $\Delta(r)$ is the nonextremality warp factor with the boundary conditions

$$\Delta(r)\Big|_{r=r_0} = 0, \quad \Delta(r)\Big|_{r=\infty} = 1, \quad (3.27)$$

where we take the horizon to be at $r = r_0$ and the boundary to be at $r = \infty$. Given (3.26), the linear combination of Ricci components, (3.22), gives rise to

$$\nabla^2 \Delta + 8\Omega_2^{-1} \nabla \Omega_2 \nabla \Delta = 0. \quad (3.28)$$

Assuming the radial dependence as above, we find the first integral of (3.28) to be

$$\frac{d\Delta}{dr} \tilde{g}_{rr}^{-1/2} \tilde{g}_5^{1/2} = A \Omega_2^{-8}, \quad (3.29)$$

where A is an integration constant, related to the temperature as we explain below. Here we have decomposed the $q = 6$ dimensional metric \tilde{g}_{mn} as follows

$$g_{mn}(y) dy^m dy^n = \tilde{g}_{rr} dr^2 + \tilde{g}_{5ij} dy^i dy^j. \quad (3.30)$$

It is easy to see now that the expression (2.2) reduces to

$$\begin{aligned} \mathcal{D} &= \frac{\sqrt{-G(r)}}{\sqrt{-G_{tt}(r)G_{rr}(r)}} \Big|_{horizon} \int_{r_0}^{\infty} \frac{\Delta d\Delta}{A} \\ &= \frac{\sqrt{-G(r)}}{\sqrt{-G_{tt}(r)G_{rr}(r)}} \Big|_{horizon} \frac{1}{2A} \\ &= \frac{1}{2A} \Omega_2^8(r_0) \tilde{g}_5^{1/2}(r_0). \end{aligned} \quad (3.31)$$

Furthermore, note that using (3.29) near the horizon yields

$$ds_{10}^2 = -(dt)^2 \Omega_2^2(r_0) \Delta^2 + \Omega_2^2(r_0) g_{rr}(r_0) \left(\frac{d\Delta}{A} \right)^2 g_{rr}^{-1}(r_0) \tilde{g}_5(r_0) \Omega_2^{16}(r_0), \quad (3.32)$$

from which we can read off the temperature

$$T = \frac{A}{2\pi \tilde{g}_5(r_0)^{1/2} \Omega_2^8(r_0)}. \quad (3.33)$$

Combining (3.31) and (3.33), we arrive at

$$\mathcal{D} = \frac{1}{4\pi T}. \quad (3.34)$$

This proves our claim that the KSS bound is always saturated in the supergravity dual, at least to this leading order in corrections.

4 A comment on α' corrections

The explicit examples in previous section, and the general theorem in section 3.4, demonstrate that the KSS bound, (1.2), is saturated in all supergravity backgrounds realizing holographic duals to gauge theories at large (strictly speaking infinite) 't Hooft coupling $\lambda \equiv g_{YM}^2 N$. On the other hand, as pointed out in [6], for typical matter (*i.e.*, water under normal conditions) $4\pi\mathcal{D}T \gg 1$. These two observations together make us conjecture that

$$\mathcal{D} = \frac{f(\lambda)}{4\pi T}, \quad (4.1)$$

where $f(\lambda)$ is (in principle) a computable function of the 't Hooft coupling, such that for arbitrary λ ,

$$f(\lambda) \geq 1, \quad (4.2)$$

and

$$f(\lambda) \rightarrow 1_+, \quad \lambda \rightarrow \infty. \quad (4.3)$$

Since on the supergravity side of the gauge/string correspondence, 't Hooft coupling corrections translate into string theory α' -corrections, verification of the conjecture (4.3) would involve the study of α' corrections to the hydrodynamics. Realistically, this can be done for the near-extremal D3-branes. In fact, using the KSS expression for the diffusion coefficient, (2.2), applied to the α' -corrected metric of the near-extremal D3-branes [19], we found that the bound (1.2) is *violated*

$$\mathcal{D}T = \frac{1}{4\pi} \left(1 - 15\gamma + \mathcal{O}(\gamma^2) \right), \quad (4.4)$$

where

$$\gamma = \frac{1}{8} \xi(3) \alpha'^3. \quad (4.5)$$

We would like to emphasize, however, that this result assumes that the dispersion relation for the low-energy gravitational shear perturbations (which led to the diffusion

coefficient expression (2.2)) is not modified by the α' corrections. This assumption is very likely incorrect, and one thus has to do complete analysis of the metric fluctuations themselves. We hope to return to this problem in the future.

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